

Exercise 27

For the following exercises, use synthetic division to find the quotient.

$$(9x^3 - x + 2) \div (3x - 1)$$

Solution

Synthetic division only works if the divisor is of the form $x - k$. Multiply the numerator and denominator by $1/3$ to make it so.

$$\frac{9x^3 - x + 2}{3x - 1} = \frac{9x^3 - x + 2}{3x - 1} \times \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{3x^3 - \frac{1}{3}x + \frac{2}{3}}{x - \frac{1}{3}}$$

Solving $x - \frac{1}{3} = 0$ gives $x = \frac{1}{3}$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & 3 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & 3 & 1 & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{c|cc|cc} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & 1 & & \\ \hline & 3 & 1 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{c|cc|cc} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & 1 & \frac{1}{3} & \\ \hline & 3 & 1 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{c|cc|cc} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & 1 & \frac{1}{3} & \\ \hline & 3 & 1 & 0 & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & 1 & \frac{1}{3} & 0 \\ \hline & 3 & 1 & 0 & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{1}{3} & 3 & 0 & -\frac{1}{3} & \frac{2}{3} \\ \hline & & 1 & \frac{1}{3} & 0 \\ \hline & 3 & 1 & 0 & \frac{2}{3} \\ \hline & & & & 3 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $3x^2 + x$.

$$\begin{aligned} (9x^3 - x + 2) \div (3x - 1) &= 3x^2 + x + \frac{\frac{2}{3}}{x - \frac{1}{3}} \\ &= 3x^2 + x + \frac{2}{3x - 1} \end{aligned}$$